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THE EFFECT OF ATMOSPHERIC TURBULENCE ON AIR TO GROUND PHOTOGRAP--ETC(U)
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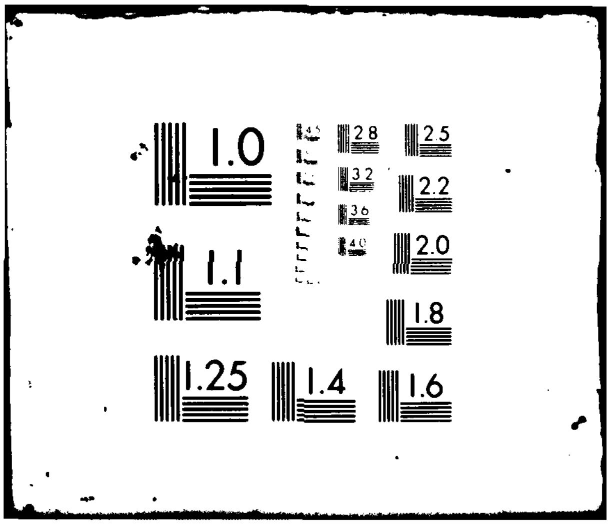
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ROYAL AIRCRAFT ESTABLISHMENT

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Technical Report 81065

May 1981

THE EFFECT OF ATMOSPHERIC TURBULENCE ON AIR TO GROUND PHOTOGRAPHY

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J.G. Ballantyne

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Procurement Executive, Ministry of Defence
Farnborough, Hants

UNLIMITED

UDC 778.35 : 551.551

ROYAL AIRCRAFT ESTABLISHMENT

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Received for printing 18 May 1981

THE EFFECT OF ATMOSPHERIC TURBULENCE ON AIR TO GROUND PHOTOGRAPHY

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SUMMARY

The current state of the theory of optical effects of atmospheric turbulence is summarized, and the derivation of an equation relating the modulation transfer function of an optical path through the atmosphere to a weighted integral of the structure constant of refractive index along that path is outlined.

Methods of estimating the structure constant from more readily available meteorological parameters are discussed, and FORTRAN subroutines for evaluating the integral and estimating the modulation transfer function of the path under any required conditions are given.

Departmental Reference: IT 175

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1 INTRODUCTION

When the optical path of an image forming system passes through a fluid medium, the random variations in its refractive index arising from turbulence modify the image formed in three main ways.

(1) The total light flux in the image of a point source varies with time.

(2) The image of the point is spread out into a patch of light.

(3) This patch moves about randomly in the image plane.

The first of these effects produces the scintillation of the stars, but is not usually of importance when imaging extended sources. The second is the most important when photographing terrestrial objects with short exposure times, but with longer exposures, the motion of the aerial image also contributes to the spread of the recorded image.

In aerial photography two types of turbulence are of importance, the local effects produced by the aircraft itself as it passes through the air, and those occurring throughout the whole of the optical path from meteorological causes. Either of these can limit the performance of an imaging system, but the former is highly dependent on the precise location of the camera in the aircraft, and on its local and overall aerodynamic design. Every case must be considered individually and experimental measurement on an actual installation is probably the only satisfactory way of determining the magnitude of the effect. In this Report we will be concerned solely with the meteorological effects.

Two distinct types of problem are involved, estimating the variations of refractive index along the optical path, and computing the effect of these on the quality of the image formed by an optical system at the end of that path. Both of these present formidable difficulties.

Experimental measurements of the turbulence in the free atmosphere have been published by a numer of workers[1-9]. These show that the magnitude of the effect can vary very rapidly in both space and time, and it seems unlikely that it will ever be possible to make accurate predictions for a given place and instant. For the purposes of system performance estimation and optimization, however, all that are required are estimated rms values for a given general area and season. It may ultimately be possible to base such estimates on statistical data on actual turbulence measurements, but until a sufficiently large data bank has been accumulated for the purpose, the best that can be done is to base the estimates on those meteorological parameters for which systematic records are generally available.

The optical effects of atmospheric turbulence are still the subject of intensive theoretical study. The classical approach of Tatarski[10] involves a number of assumptions and approximations whose validity has been discussed, *inter alia* by Strohbehn[11], but as will be shown, some of these limitations are not too serious in our particular application, and any bias introduced by the others is probably small compared with the errors arising from uncertainties in the refractive index data. We have therefore restricted our treatment to the classical theory, and an outline of the derivation of an expression

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for the modulation transfer function of a turbulent optical path will be given in the following section.

2 A SUMMARY OF THE THEORY

2.1 Basic concepts

The optical effects of turbulence arise from the random variations in space and time of the refractive index of the air or other medium through which the light passes on its way from the object to the sensor. An excellent and up to date introduction to the subject has been given by Hufnagel[1], and the treatment given here will be based closely on this.

The variation in space of a random variable R may be described statistically by a structure function

$$D_R(\underline{r}) = \left\langle [R(\underline{r}_1) - R(\underline{r}_2)]^2 \right\rangle \qquad (1)$$

where $\underline{r}_1$ and $\underline{r}_2$ are vectors defining two points in space,

and $\underline{r} = \underline{r}_1 - \underline{r}_2$.

The angle brackets denote the ensemble average.

D_R is related to the more familiar parameters σ_R , the variance and ρ_R , the auto correlation function, by the equation

$$D_R(\underline{r}) = 2\sigma_R^2 \{1 - \rho_R(\underline{r})\} \quad .$$

In the Kolmogerov model of turbulence in an isotropic medium the structure function takes the form

$$D_R(r) = C_R^2 r^{\frac{2}{3}} \quad . \qquad (2)$$

The constant of proportionality C_R^2 is known as the structure constant of R . It depends on the local properties of the atmosphere, and may vary along the optical path. Clearly the model must break down for very large r as, in a macroscopically homogeneous medium, D_R will tend to a limit (twice the variance) as r increases. We will return to this point later.

The Kolmogorov model can also be expressed in terms of the spectral density function of R .

$$\Phi_R(K) = \frac{5\sqrt{3}\Gamma(2/3)}{36\pi^2} K^{-\frac{11}{3}} C_R^2$$

$$\simeq 0.033 K^{-\frac{11}{3}} C_R^2 \qquad (3)$$

where K is the spatial frequency.

We will also make use of F_R the two-dimensional spectral density of the structure function of R which is related to it by the integral transform

$$D_R(\rho) = 4\pi \int_0^\infty \left[1 - J_0(\kappa\rho)\right] F_R(\kappa) \, . \, d\kappa \tag{4}$$

where ρ and κ are position and frequency vectors in a plane.

The central problem is to relate the structure function of the complex amplitude of a wave front at the aperture of the detecting system, to that of the refractive index of the air or other medium along the path from the object to the detector.

The method used was developed by Tatarski, who has given a very full treatment of the plane wave case[1]. In normal terrestrial photography, however, we are concerned with spherical wave fronts, but these can be treated in a very similar way, as indicated, for example, by Carlson and Ishimaru[12].

2.2 The wave equation

The propagation of light through a turbulent medium can be expressed as a set of scalar wave equations of the form

$$\nabla^2 u + k^2 n^2(\underline{r}) u = 0 \tag{5}$$

where u is any component of the electromagnetic field

k is the wave number

$n(\underline{r})$ is the refractive index at the point in space represented by the vector $\underline{r}$.

It is convenient to express u in terms of its log amplitude and phase, and to express these as the sum of a solution to the unperturbed equation and small perturbations to the solution arising from the random departures of n from its mean value (assumed unity).

Let

$$u = Ae^{iS} \tag{6}$$

$$\psi = \ln u = \ln A + iS \tag{7}$$

$$n = 1 + n_1 \tag{8}$$

$$u = u_0 + u_1 \tag{9}$$

$$\psi = \psi_0 + \psi_1 \, . \tag{10}$$

Substituting these in equation (5) we obtain

$$\nabla^2 \psi_1 + 2\nabla\psi_0 \, . \, \nabla\psi_1 + 2k^2 n_1(\underline{r}) = 0 \tag{11}$$

which has the solution

$$\psi_1(\underline{r}) = \frac{k^2}{2\pi u_0(\underline{r})} \int_V \frac{n_1(\underline{r}')u_0(\underline{r}') \exp\left(ik|\underline{r} - \underline{r}'|\right)}{|\underline{r} - \underline{r}'|} \, dV' \tag{12}$$

where the integration is carried out throughout the entire space occupied by the medium, and $\underline{r}'$ is a vector representing a point in that space.

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For a spherical wave

$$u_0(\underline{r}) = U \exp(ikr)/r \tag{13}$$

and

$$\psi_1(\underline{r}) = \frac{k^2 r}{2\pi}\int_V \frac{n_1(\underline{r}') \exp[ik(r' - r)] \exp\left[ik|\underline{r} - \underline{r}'|\right]}{r'|\underline{r} - \underline{r}'|} dV' \ . \tag{14}$$

Resolving the r into components x , along, and y and z perpendicular to the line of sight, and neglecting second order terms in y and z , this reduces to

$$\psi_1(\underline{r}) = \frac{k^2 x}{2\pi}\int_V \frac{n_1(\underline{r}') \exp\left\{ik \dfrac{2xx'(yy' + zz') - x'^2(y^2 + z^2) - x^2(y'^2 + z'^2)}{x'(x - x')}\right\}}{x'(x - x')} dV' \ . \tag{15}$$

Multiplying these equations by their complex conjugates and taking the ensemble average, we obtain - after considerable manipulation - the following relations between the two-dimensional spectra of the phase and amplitude structure functions in a plane perpendicular to the optical path, at its far end, and the refractive index variations along that path.

$$F_A(K) = 2\pi k^2 \int_0^L \left(\frac{L}{\eta}\right)^2 \sin^2\left[\frac{L(L - \eta)K^2}{2kK}\right] C_n^2(\underline{\eta})\Phi_n^{(0)}\left(\frac{KL}{\eta}\right) d\eta \ . \tag{16}$$

$$F_S(K) = 2\pi k^2 \int_0^L \left(\frac{L}{\eta}\right)^2 \cos^2\left[\frac{L(L - \eta)K^2}{2kK}\right] C_n^2(\underline{\eta})\Phi_n^{(0)}\left(\frac{KL}{\eta}\right) d\eta \ , \tag{17}$$

where η is distance measured along the optical path

L is its total length

$\Phi_n^{(0)}$ is the value of Φ_n for $C_n^2 = 1$.

The amplitude and phase structure functions may be computed from these expressions, by means of equation (4), but a much simpler expression is obtained if, instead of evaluating these individually, we calculate their sum, which is known as the wave structure function.

$$D_W(\rho) = 4\pi \int_0^\infty \left[1 - J_0(K\rho)\right]\left[F_A(K) + F_S(K)\right] K dK$$

$$= 8k^2\pi^2 \int_0^\infty \left[1 - J_0(K\rho)\right] K \int_0^L C_n^2(\underline{\eta}) \left(\frac{L}{\eta}\right)^2 \Phi_n^{(0)}\left(\frac{KL}{\eta}\right) d\eta dK$$

$$= \frac{10\sqrt{3}\Gamma(2/3)k^2}{9} \int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\eta) \int_0^\infty [1 - J_0(K\rho)]K^{-\frac{8}{3}} dK d\eta$$

$$= \frac{\Gamma(2/3)\Gamma(1/6)}{3^{\frac{1}{2}}2^{\frac{5}{3}}\Gamma(11/6)} k^2 \rho^{\frac{5}{3}} \int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\eta) d\eta$$

$$\simeq 2.914 k^2 \rho^{\frac{5}{3}} \int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\eta) d\eta \quad . \tag{18}$$

In our application, ρ is very small compared with L and the variation in amplitude across the aperture is small compared with the variation in phase, so D_W may be used in place of D_S with little error. Further, there is some experimental evidence[1,2,13] that phase effects saturate less quickly than those involving amplitude or intensity, so the Kolmogorov approximation is more nearly valid.

2.3 Modulation transfer function for long exposures

The imperfections in the image produced by a lens may be described in terms of its amplitude spread function which may be calculated from the Fourier transform of the complex amplitude of the wave front at its exit pupil.

It follows that the intensity spread function is given by the Fourier transform of the autocorrelation function of the complex amplitude in the exit pupil, and that the autocorrelation function itself, suitably normalized, gives the modulation transfer function of the lens, see for example Ref 14.

A parallel result may be obtained in the case of the imperfections in the image formed by the lens as a result of the random disturbances, due to turbulence, in the wave front arriving at the entrance pupil of the lens, and it may be shown[15] that the modulation transfer function, averaged over a long period of time, arising from this cause is simply the spatial coherence function in the pupil, *ie*

$$\langle M_\ell(f)\rangle = \langle E\rangle^{-1} \langle \mathscr{E}(\underline{r} + F\lambda f)\mathscr{E}^*(\underline{r})\rangle \quad . \tag{19}$$

where F is the focal length of the lens

λ is the wave length of the light

f is the spatial frequency in cycles/unit distance

$\mathscr{E}(r) = \exp\left[ikr - i\omega t + A(\underline{r},t) + iS(\underline{r},t)\right]\langle E(r)\rangle^{\frac{1}{2}}$

hence

$$\langle M_\ell(f)\rangle = \left\langle \exp\left[A(\underline{r} + F\lambda f, t) + A(\underline{r}) + iS(r + F\lambda f) - iS(r)\right]\right\rangle \quad .$$

The quantity in square brackets is normally distributed with mean $2\langle A\rangle$, the variance of the real part is $2\sigma_A^2\{1 + \rho_A(F\lambda f)\}$ and the variance of the imaginary part is $2\sigma_S^2\{1 - \rho_S(F\lambda f)\}$ where ρ_A and ρ_S are the amplitude and phase autocorrelation functions,

hence

$$\langle M_\ell(f)\rangle = \exp\left[2\langle A\rangle + 2\sigma_A^2\left\{1 + \rho_A(F\lambda f)\right\} - 2\sigma_S^2\left\{1 - \rho_S(F\lambda f)\right\}\right] 2\langle A\rangle + 2\sigma_A^2 \equiv 0 \quad .$$

and

$$\langle M_\ell(f)\rangle = \exp\left[-2\sigma_A^2 + 2\sigma_A^2\left\{1 + \rho A(F\lambda f)\right\} - 2\sigma_S^2\left\{1 - \rho(F\lambda f)\right\}\right] \quad .$$

By making use of the properties of log-normal random variable (see *eg* Ref 1, section 6.9) this can be reduced to

$$\begin{aligned} \langle M_\ell(f)\rangle &= \exp\left[-\sigma_A^2\left\{1 - \rho_A(F\lambda f)\right\} - \sigma_S^2\left\{1 - \rho_S(F\lambda f)\right\}\right] \\ &= \exp\left[-\tfrac{1}{2}D_A(F\lambda f) - \tfrac{1}{2}D_S(F\lambda f)\right] \\ &= \exp\left[-\tfrac{1}{2}D_W(F\lambda f)\right] \quad . \end{aligned} \tag{20}$$

This expression represents the image degradation due to turbulence when the exposure time is long compared with period of the turbulence, and includes the effects of both the instantaneous spread of the image, and its movement in the focal plane.

$\langle M_\ell(f)\rangle$ may be expressed in a dimensionless form as

$$\langle M_\ell(f)\rangle = \exp\left(-f_t^{\frac{5}{3}}\right)$$

where $f_t = f/f_T$

and $$f_T = 0.0879\left(\lambda^{\frac{1}{5}}/F\right)\left\{\int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\eta)\,d\eta\right\}^{-\frac{3}{5}} \quad .$$

The form of the function is shown in Fig 1a, together with a Gaussian distribution which has the same value at $f_t = 1$ for comparison. The two curves are superficially somewhat similar in appearance, but there is one important difference: the second derivative of the turbulence curve is infinite at zero frequency. The consequences of this will be mentioned later.

An interesting limiting case occurs when the upper limit of the integral is extended into a region in which C_n^2 becomes negligible. Under these circumstances, the integral becomes proportional to $L^{-\frac{5}{3}}$, so f_T is proportional to L for a given angle of path, and the scale of the MTF function, expressed in cycles per unit length at the target is independent of L .

2.4 Modulation transfer function for short exposures

The image spread averaged over long periods may be regarded as the convolution of the instantaneous spread with the probability distribution of the position of its centroid.

Using the Fourier shifting theorem, M_s the modulation transfer function corresponding to spread relative to the instantaneous centroid of the image, which is displaced by $\underline{\alpha}$ from its mean position, is related to the long period average MTF by

$$M_\ell(f) = M_s(f)\exp\left[-\,2\pi i\underline{\alpha}\,.\,\underline{f}\right]$$

and the mean values by

$$\begin{aligned}\langle M_\ell(f)\rangle &= \langle M_s(f)\rangle\langle \exp(-\,2\pi i\underline{\alpha}\,.\,\underline{f})\rangle\\ &= \langle M_s(f)\rangle\exp\left[2\pi^2\langle\underline{\alpha}\,.\,\underline{f}\rangle\right]\\ &= \langle M_s(f)\rangle\exp\left[-\,\pi^2 f^2\langle\underline{\alpha}\,.\,\underline{\alpha}\rangle\right]\end{aligned}$$

$\underline{\alpha}$ is given by the mean rate of change of phase over the pupil, multiplied by F/k, so in the one dimensional case

$$\langle\underline{\alpha}\,.\,\underline{\alpha}\rangle = (F/pk)^2 D_S(p)$$

where p is the width of the pupil.

In two dimensions, we would expect the value to be approximately twice this, and it can be shown[1] that for a clear circular aperture of diameter d

$$\begin{aligned}\langle\underline{\alpha}\,.\,\underline{\alpha}\rangle &= 2\times\frac{25\times 2^{\frac{1}{3}}\,\Gamma\left(\frac{5}{3}\right)}{33\left[\Gamma\left(\frac{11}{6}\right)\right]^2}\left(\frac{F}{dk}\right)^2 D_S(d)\\ &= 2\times 0.975\left(\frac{F}{dk}\right)^2 D_S(d)\quad .\end{aligned}$$

The correcting factor is normally taken to be unity so

$$\langle M_\ell(f)\rangle = \langle M_s(f)\rangle\exp\left[-\,\pi^2 f^2\, 2(F/dk)^2 D_S(d)\right]$$

re-writing in terms of the cut off frequency of the lens

$$f_0 = kd/2\pi F$$

$$\begin{aligned}\langle M_\ell(f)\rangle &= \langle M_s(f)\rangle\exp\left[-\,\tfrac{1}{2}(f/f_0)^2 D_S(d)\right]\\ &= \langle M_s(f)\rangle\exp\left[-\,\tfrac{1}{2}(f/f_0)^{\frac{1}{3}} D_S(F\lambda f)\right]\end{aligned}$$

and since

$$D_S \triangleq D_W$$

$$\langle M_s(f)\rangle \triangleq \exp\left[-\,\tfrac{1}{2}\left\{1-(f/f_0)^{\frac{1}{3}}\right\}D_W(F\lambda f)\right]\quad . \tag{21}$$

$M_s(f)$, unlike $M_\ell(f)$ depends on the aperture of the lens being used to collect the radiation, so it cannot strictly be regarded as a transfer function for the optical path, but only as a multiplying factoı .. ıe ɛ ıied to the MTF of the lens. It has no significance for values of $f > f_0$ where the MTF of the lens is identically zero.

Equation (21) may be expressed in the form

$$\langle M_s(f) \rangle = \exp\left\{- 57.52T\left(1 - f_r^{\frac{1}{3}}\right) f_r^{\frac{5}{3}}\right\}$$

where $f_r = f/f_0$

and T is a dimensionless parameter

$$T = (d^{\frac{5}{3}}/\lambda^2) \int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\eta) d\eta$$

all the lengths being expressed in the same units.

The minimum value of $\langle M_s(f) \rangle$ occurs at

$$f_r = \left(\frac{5}{6}\right)^3 = 0.5787$$

and has the value

$$M_{min} = \exp(- 3.853T) \quad .$$

This is probably the most convenient single parameter for expressing the effect of turbulence in short exposures.

The form of $\langle M_s(f) \rangle$ for a range of values of T is shown in Fig 2 together with the corresponding values of $\langle M_\ell(f) \rangle$ for comparison.

2.5 Intermediate exposures

The way in which the form of the turbulent "modulation transfer function" varies with exposure time depends on the specific conditions. Hufnagel[1] states that for photographic systems with exposure times longer than several milliseconds image motion contributes significantly to the overall blur. On the other hand, Roddier and Roddier[16], who were concerned primarily with astronomical images, found that the difference between the image power spectra at zero and 0.02 second exposure time is negligible, and even at 0.1 second exposure the difference is small except close to the cut off frequency. Further work is required on this problem, but as the exposure times used in air to ground photography are of the order of a very few milliseconds, the short exposure form, equation (21) may usually be employed.

2.6 Line spread function

A line spread function representing the effect of turbulence in long exposures may be obtained by Fourier transforming equation (20), and is shown in Fig 1b as a function of

$$x_t = f_T x \quad .$$

It falls off very slowly at large distances, and because the function transformed has an infinite second derivative at zero frequency, its standard deviation is infinite. If a

characteristic dimension is required, the quartile

$$x_q \approx 0.150/f_T$$

or the standard deviation of the equivalent Guassian distribution

$$\sigma_G = 1/(\sqrt{2}\pi f_T) \simeq 0.225/f_T$$

may be used. Other linear parameters may be obtained from Fig 1b. The effects of turbulence in short exposures cannot be expressed directly in terms of a line spread function because the Fourier transform of equation (21) does not converge, but the combined line spread function of a turbulent path, and the receiving lens, may be computed by Fourier transforming the product of their modulation transfer functions.

The combined MTFs of a diffraction limited lens, and a number of turbulent optical paths are shown in Fig 3, and the corresponding line spread functions are plotted against

$$x_r = f_0 x$$

in Fig 4. The corresponding long exposure cases are also shown for comparison.

3 ESTIMATING THE STRUCTURE CONSTANT OF REFRACTIVE INDEX

3.1 General approach

The way in which C_n^2 varies with place and time has been measured by a number of workers[1-9]. As might perhaps be expected, the variations are large and erratic, and there would seem no prospect of predicting them in detail for some point in space on a given occasion.

For the purposes of estimating the performance of reconnaissance systems, and of optimizing this, however, all we require are weighted average values along an optical path for a given time of day, season, altitude and broadly specified geographical area and weather situation. It may, in time, be possible to accumulate sufficient statistical data on C_n^2 to estimate these directly, but in view of the difficulty of the measurements this seems a rather remote possibility, and for the present we will have to estimate the mean values of C_n^2 indirectly from meteorological parameters for which detailed statistics are generally available.

Such estimates are unlikely to be very accurate, but the large random errors which will occur will probably swamp any bias introduced, and they should be adequate for broad system design work.

The dominant meteorological factors involved differ at high and low altitudes and the two cases will be discussed separately.

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3.2 Lower atmosphere

The local variations in the refractive index of the atmosphere in turbulence arise primarily from the variations in temperature, and are related to them by the equation

$$\frac{dn}{dT} = 78 \times 10^{-6} \frac{P}{\mathcal{T}^2} \quad , \tag{22}$$

where P is the local pressure in mb

$\mathcal{T}$ is the absolute temperature.

For rough statistical work, values of P and $\mathcal{T}$ based on the ICAO standard atmosphere[17] may be used.

In the lowest layers of the atmosphere turbulence is primarily due to insolation. It is therefore convenient, first to estimate the structure constant of temperature, and then to derive the structure constant of refractive index from it using equation (22) and the relation

$$C_n^2 = \left(\frac{dn}{d\mathcal{T}}\right)^2 C_{\mathcal{T}}^2 \quad . \tag{23}$$

A number of formulae have been given for the variation of $C_{\mathcal{T}}^2$ with altitude in the boundary layer. The simplest of these is a negative four thirds power law[18,10]

$$C_{\mathcal{T}}^2 \propto h^{-\frac{4}{3}} \quad . \tag{24}$$

However, this must break down at very low altitudes as not only does it give infinite values of $C_{\mathcal{T}}^2$ and C_n^2 at zero height, but the weighted integral of C_n^2 from zero to any finite height is also infinite.

Wyngaard *et al*[19] have given a modified formula which varies as $h^{-\frac{2}{3}}$ at very low height, but reverts to $h^{-\frac{4}{3}}$ further up, the transition taking place at a height of a few metres. This expression is also infinite at zero height, but the integral remains finite, and tends to zero as the path of integration tends to zero.

A convenient form of the formula has been given by Hufnagel[1].

$$C_{\mathcal{T}}^2(h) = \frac{2 \times 10^{-3} Q^{\frac{4}{3}} h^{-\frac{4}{3}}}{\left[1 + \dfrac{1400 u_*^3}{hQ}\right]^{\frac{2}{3}}} \tag{25}$$

where Q is the upward convective heat flux $W\ m^{-2}$

h is the height in metres

u_* is a characteristic frictional velocity $m\ s^{-1}$.

If Q and u_* can be estimated, C_n^2 can be evaluated for each point on the optical path using equations (23) and (25), and D_w , the wave structure function at the end of the path computed by the integrating of equation (18). This integration must be performed for each Q , u_* and path configuration.

Q is dependent on such factors as solar elevation, cloud cover and the type of terrain. As a rough guide[1] we may take

$$Q = Q_0 \sin \zeta - 50 \tag{26}$$

where ξ is the solar elevation and typical values of Q_0 are as shown in Table 1. When the right hand side of equation (26) is negative, Q may be taken as zero.

Table 1

Q_0 in watts per metre^2

Type of terrain \ Sky condition	Continuously clear	Overcast
Dry sand or lava	500	200
Dry fields of brush	400	150
Wet fields	200	70

For intermediate amounts of cloud we may estimate Q_0 by linear interpolation. The use of the words 'continuously clear' in Table 1 should be noted, as changes in turbulence always lag somewhat behind the meteorological factors producing them. A similar slight asymmetry will occur between morning and evening conditions.

u_* is typically an order of magnitude smaller than the local wind speed. Its effect on C_T^2 is only significant at low heights, and its contribution to the weighted integral even smaller, if the detector is well above the transition point as will normally be the case in air to ground reconnaissance. For most of our applications we have therefore used a mean value of u_*^3/Q which gives a break point between the two forms at a height of a few metres, say three, and written

$$C_T^2(h) = \frac{2 \times 10^{-3} Q^{\frac{4}{3}} h^{-\frac{4}{3}}}{\left[1 + \frac{3}{h}\right]^{\frac{2}{3}}} \quad . \tag{27}$$

This has the great advantage that, the integral in equation (18) can be expressed as a product of $Q^{\frac{4}{3}}$ and a function of height which may be pre-computed and fitted by an empirical function for routine use as will be described in section 4.2.

3.3 Upper atmosphere

Above the boundary layer, the mean value of C_n^2 falls off rather more slowly than at low levels, though a sharp local maximum occurs in the neighbourhood of the tropopause, which is often a region of high wind shear.

Hufnagel[1,20] has given an empirical formula expressing the structure constant of refractive index in the region above the first strong temperature inversion as a function of height and of the root mean square wind speed averaged over the height band 5-20 km.

$$C_n^2(h) = 8.2 \times 10^{-56} U^2 h^{10} \exp(-h/1000) + 2.7 \times 10^{-16} \exp(-h/1500) m^{-\frac{2}{3}} \tag{28}$$

where h is in metres

U is the average rms wind speed in the height band 5-20 km in metres per second.

Values of U for any given area and season may be deduced from the maps in "Upper Winds over the World"[21]. Hufnagel suggests 27 m s^{-1} as typical, but this appears to have been based on experiments in an area of higher than average upper winds.

For U = 27 , equation (28) gives a peak value at h = 11 km of about 10^{-17} $m^{-\frac{2}{3}}$. This is numerically quite close to the peak value shown in Fig 5 of Hufnagel's earlier paper[22] and Fig 6 of Hufnagel and Stanley's paper[15], but as these are expressed in units of cm they represent a much larger turbulence. However, as far as can be deduced from the small scale graphs, the peak is also very much narrower than that given by equation (28), and it may be that the explanation of this discrepancy is that the earlier graph represents a typical profile, while equation (28) represents an ensemble average of all occasions, and the peak is therefore flattened by the variations in the height of the tropopause.

3.4 Intermediate heights

The form of the transition between the two types of turbulence structure is highly dependent on the specific weather situation, but unless it is possible to forecast the thickness of the boundary layer, the best that can be done is to flare the curves into each other in the transition region, so as to approximate to which ever is the greater.

The simple form

$$C_N^2 = \left(C_N^2\right)_{low} \exp(-\ 0.00001H^2) + \left(C_N^2\right)_{high} \tag{29}$$

fits the data with a peak error of less than 5% for a Q of 100. There is no need to attenuate the $\left(C_N^2\right)_{high}$ term at low altitudes as it becomes small compared with $\left(C_N^2\right)_{low}$ unless both are negligible.

$C_N^2(H)$ may be expressed as

$$C_N^2(H) = C_1(H) + C_2(H)U^2 + C_3(H)Q^{\frac{4}{3}}$$

and the forms of the coefficients C_1, C_2 and C_3 are shown in Fig 5.

4 COMPUTATIONAL FORMULAE

4.1 Modulation transfer function

The expressions for long and short exposure modulation transfer function can be combined in the form

$$M(f) = \exp\left[-\ 2.914(2\pi^2)\left\{1 - s\left(\frac{F\lambda f}{d}\right)^{\frac{1}{3}}\right\}\lambda^{-\frac{1}{3}}(Ff)^{\frac{5}{3}}I_L\right]$$

where $I_L = \int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\eta)\,d\eta$

s is 0 for slow shutter speeds

1 for fast shutter speeds.

Rewritten in terms of more convenient units we have

$$M(f_{mm}) = \exp\left[-57528\left\{1 - 0.01s\left(N\lambda_{nm}f_{mm}\right)^{\frac{1}{3}}\right\}\lambda_{nm}^{-\frac{1}{3}}\left(F_{mm}f_{mm}\right)^{\frac{5}{3}}I_L\right]$$

where N is the f/number of the lens

F_{mm} is the focal length of the lens in millimetres

λ_{nm} is the wave length of the light in nanometres

f_{mm} is spatial frequency in cycles per millimetre

or in angular terms

$$M(\omega_{mr}) = \exp\left[-0.5753 \times 10^{10}\left\{1 - 0.1s\left(\frac{\lambda_{mm}\omega_{mr}}{d_{mm}}\right)^{\frac{1}{3}}\right\}\lambda_{nm}^{-\frac{1}{3}}\omega_{mr}^{\frac{5}{3}}I_L\right]$$

where ω_{mr} is the spatial frequency in cycles per milliradian

d_{mm} is the diameter of the lens in millimetres.

The latter form is used in the subroutine to be described, as it involves one fewer variable.

4.2 Evaluation of the integral

Consider an optical path from a target at height H_0 to a camera at height H_1 at a plan range of R_p (see Fig 6). Then if we assume that over this optical path C_n^2 may be treated as a function of height only,

$$I_L = \int_0^L \left(\frac{\eta}{L}\right)^{\frac{5}{3}} C_n^2(\underline{\eta})\,d\eta$$

$$= \sqrt{(H_1 - H_0)^2 + R_p^2}\int_0^1 \xi^{\frac{5}{3}} C_n^2\left\{H_0 + \xi(H_1 - H_0)\right\}d\xi$$

where $\xi = \eta/L$.

For paths with a small height range the integral may be approximated by one of the following generalized Gauss quadrature formulae[23].

$$I_L = \sqrt{(H_1 - H_0)^2 + R_p^2}\,\frac{3}{8}\,C_n^2\left(\frac{3}{11}H_0 + \frac{8}{11}H_1\right)$$

$$I_L = \sqrt{(H_1 - H_0)^2 + R_p^2}\left\{\begin{array}{l}0.1195C_n^2(0.574H_0 + 0.426H_1) \\ + \\ 0.2555C_n^2(0.132H_0 + 0.868H_1)\end{array}\right\}$$

which are exact if C_n^2 varies linearly, or as a cubic in H respectively, over the required interval.

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For wider ranges of H the weighted integral becomes complex in form and is best evaluated piecewise using variable step length.

For paths starting at the ground we may write

$$I_L = \sqrt{1 + (R_p/H_1)^2}\, I_H$$

where $$I_H = \int_0^{H_1} \left(\frac{h}{H_1}\right)^{\frac{5}{3}} C_n^2(h)\,dh \quad .$$

This special case is of very frequent occurrence, and to save time in routine analysis the integral I_H has been pre-computed for $1 < H_1 < 10^5$ metres, and fitted by an empirical function.

$$I_{H_1} = I_1(H_1) + U^2 I_2(H_1) + Q^{\frac{4}{3}} I_3(H_1) \quad .$$

Simple polynomial fits to I_1, I_2 and I_3 were not successful, but it was found that their logarithms could be expressed as the difference of a polynomial, and a root of a polynomial in log H_1. The coefficients of the two polynomials for each function were determined by the iterative use of the ICL curve fitting sub-routine F4CFORPL and are shown in Table 2, together with their mean square error weighted to take account of the relative importance of the three terms at any height. Two versions of I_2 are given. In view of the probable error in estimating U^2, the simpler form will often be adequate.

Table 2

Empirical expressions for I_1, I_2 and I_3

$$I_1 = 10\uparrow\left(-\,9.293 - 0.3977y - 0.01883y^2 - \sqrt{0.8151 - 1.805y + 1.802y^2}\right) \quad (2.5\%)$$

$$I_2 = 10\uparrow\left(-\,13.04 + 2.886y + 0.2075y^2 + 0.1520y^3 - \sqrt{60.35 - 108.0y + 59.30y^2}\right) \quad (8\%)$$

$$I_2 = 10\uparrow\left\{\begin{array}{l} -\,13.53 + 3.453y + 0.2414y^2 + 0.1045y^3 + 0.007853y^4 \\ -\,\sqrt[4]{2712 - 10009y + 15544y^2 - 11629y^3 + 3527y^4} \end{array}\right\} \quad (2\%)$$

$$I_3 = 10\uparrow\left(-\,14.20 - 0.3068Y - 0.1317Y^2 - \sqrt{1.445 - 1.177Y + 0.2405Y^2}\right) \quad (3\%)$$

where $y = \log(H_1/1000)$

$Y = \log(H_1/10)$

H_1 = height of sensor in metres .

4.3 FORTRAN subroutines

The equations described in the previous section have been implemented in four subroutines and an auxiliary function segment written in extended ICL 1900 FORTRAN, which are included in the 'PHOTRAN' subroutine library[24].

They are as follows:

SUBROUTINE TIH(H1,Q,U,TI)
which evaluates I_H

SUBROUTINE TIL(HO,H1,RP,Q,U,TI)
which evaluates I_L

SUBROUTINE CSN(H,Q,U,C)
which evaluates C_n^2

SUBROUTINE MTFT(NEW,FA,S,D,WL,HO,H1,RP,Q,U,T)
which computes the modulation transfer function.

FUNCTION CSNFUN(RSF)
which provides values of $\eta^{\frac{5}{3}} C_n^2(\eta)$ for evaluating I_L for paths which do not start at ground level.

The variables used in the CALL statements are as follows

HO	target height in metres
H1	camera height in metres
RP	plan range in metres
WL	wave length in nanometres
D	lens diameter in millimetres
S	shutter speed parameter, 0 for long exposure 1 for short exposures.

The subroutine will accept intermediate values.

NEW	is set to 1 when a new optical path is required, and to 0 when only the frequency is changed.
Q	upward convective heat flux in watts metre^{-2}
U	the average rms wind speed in the height band 5-20 km, in metres per second
FA	spatial frequency in cycles per milliradian. If FA is set to -1 , the frequency at which the short exposure 'MTF' is least will be substituted.
T	Modulation transfer function at a frequency FA
RSF	= η/L

HO, H1, Q and U are passed from TIL to CSNFUN via a common block
COMMON/TURBDATA/HO,H1,Q,U .

Detailed listings are given in the Appendix.

One point that should be noted is that if subroutine TIL is called with $H1 < 1$ metre it will be set to 1 metre and this value will be used in the computation, and returned to the calling segment at exit.

5 DISCUSSION

The theoretical difficulties and the scarcity of experimental data have been stressed throughout this Report, and there seems no immediate prospect of predicting the effect of turbulence on the image quality of an individual exposure.

The methods presented here, however, should provide a convenient, and adequately accurate way of introducing the average effect of turbulence, in some area and broadly defines meteorological situation, into mathematical models for use in the analysis and optimization of reconnaissance systems, both photographic and electro-optical. Some detailed examples of their application will be given elsewhere.

Perhaps the most important result to emerge is that there is a large difference in the performance obtainable in short and long exposures. High aperture optical systems are to be preferred, not only because they permit shorter exposures, and hence reduce or eliminate the turbulent image movement, but also because the residual static blurr is aperture dependent.

The exact form of transition between the long and short exposure cases is one field which would repay further investigation.

Acknowledgment

The author wishes to thank Dr W.T. Roach of the Meteorological Office at Bracknell, for a most helpful discussion, and for drawing his attention to a number of the references used here.

Appendix

LISTINGS OF SUBROUTINES

```
      SUBROUTINE CSN(H,Q,U,C)
      T=216.68
      P=226.24*EXP(.00015765*(11000-H))
      IF(H.GT.11000)GOTO 1
      T=288.18-.0065*H
      P=1013.2*EXP(5.25766*ALOG(1-.00002255535*H))
    1 G=(.000079*P/(T*T))**2
      C= 2.7E-16*EXP(-H/1500)
     1 +U*U*8.2E-56*H**10*EXP(-.001*H)
     1 +G*EXP(-.00001*H*H)*.002*(Q/H)**(4./3.)/(1.+3./H)**(2./3.)
      RETURN
      END


      SUBROUTINE TIH(H1,Q,U,TI)
      DATA A0,A1,A2/-9.293,-.3977,-.01883/
      DATA B0,B1,B2/.8151,-1.805,1.802/
      DATA C0,C1,C2,C3,C4/-13.53,3.453,.2414,.1045,.007853/
      DATA D0,D1,D2,D3,D4/2712,-10009,15544,-11629,3527/
      DATA E0,E1,E2/-14.2,-.3068,-.1317/
      DATA F0,F1,F2/1.445,-1.177,.2405/
      Z=ALOG10(H1)-1.
      Y=Z-2.
      TI=10**(A0+Y*(A1+Y*A2)-SQRT(B0+Y*(B1+Y*B2)))
     1+U*U*10**(C0+Y*(C1+Y*(C2+Y*(C3+Y*C4)))
     1-(D0+Y*(D1+Y*(D2+Y*(D3+Y*D4))))**(.25))
     1+Q**(4./3.)*10**(E0+Z*(E1+Z*E2)-SQRT(F0+Z*(F1+Z*F2)))
      RETURN
      END


      SUBROUTINE MTFT(NEW,FA,S,D,WL,H0,H1,RP,Q,U,T)
      IF(NEW.EQ.1)CALL TIL(H0,H1,RP,Q,U,TI)
      IF(FA.LE.-0.000001)FA=578.7*D/WL
      T=EXP(-.5753E10*(1-.1*S*(WL*FA/D)**(1./3))
     1  *WL**(-1./3.)*FA**(5./3.)*TI)
      RETURN
      END
```

```
      SUBROUTINE TIL(H0,H1,RP,Q,U,TI)
      COMMON/TURBDATA/HH0,HH1,QQ,UU
      EXTERNAL CSNFUN
      IF(H1.LT.1.)H1=1.
      HH0=H0
      HH1=H1
      QQ=Q
      UU=U
      RS=SQRT((H1-H0)**2+RP*RP)
      IF(H0.LT.0.001)GOTO 300
      CALL CSN(.574*H0+.426*H1,Q,U,C0)
      CALL CSN(.132*H0+.868*H1,Q,U,C1)
      CBB,CB=.1195*C0+.2555*C1
      IF(ABS(H1/H0-1.).GT.0.3)CALL F4INTSMP(0.,1.,CSNFUN,.001*CB,3,CBB)
      TI=CBB*RS
      RETURN
  300 CALL TIH(H1,Q,U,TH)
      TI=TH*RS/H1
      RETURN
      END

      FUNCTION CSNFUN(RSF)
      COMMON/TURBDATA/H0,H1,Q,U
      H=H0+(H1-H0)*RSF
      CALL CSN(H,Q,U,C)
      CSNFUN=RSF**(5./3.)*C
      RETURN
      END
```

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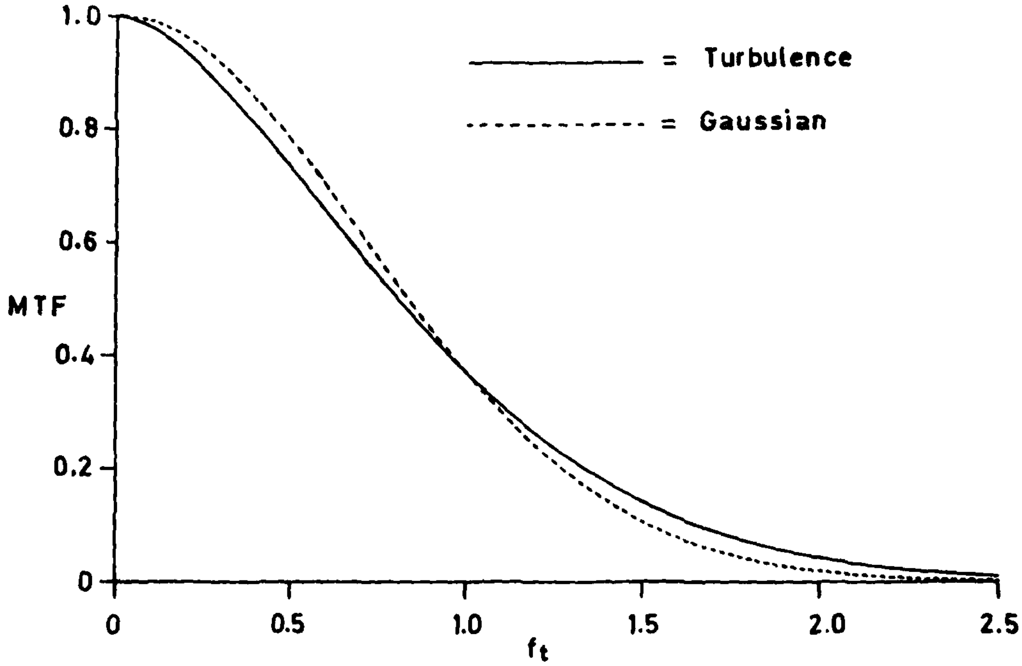

Fig 1a MTF for long exposures

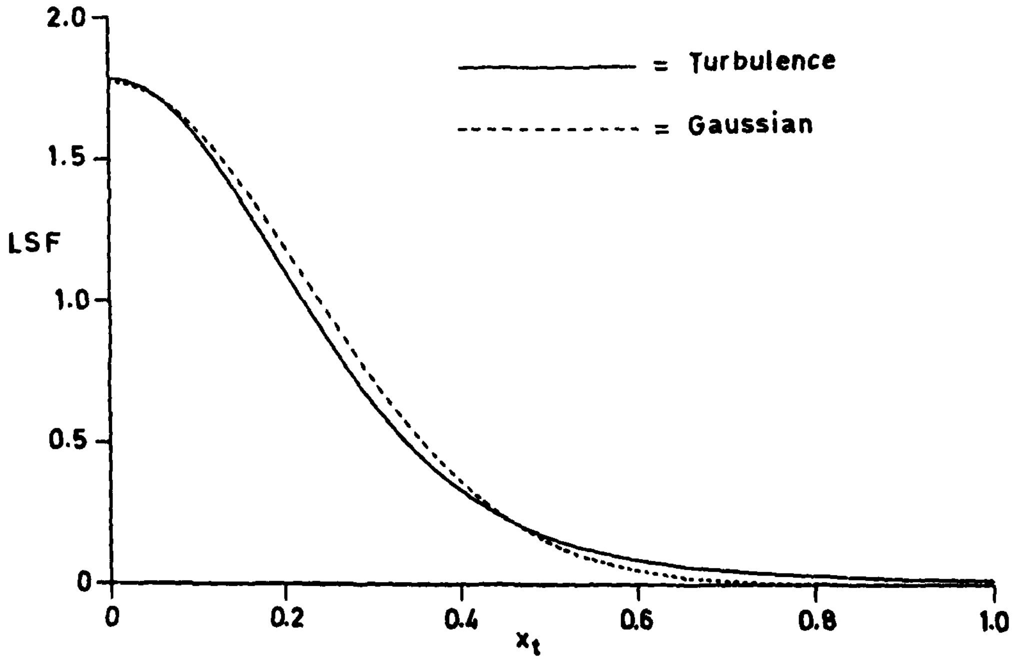

Fig 1b LSF for long exposures

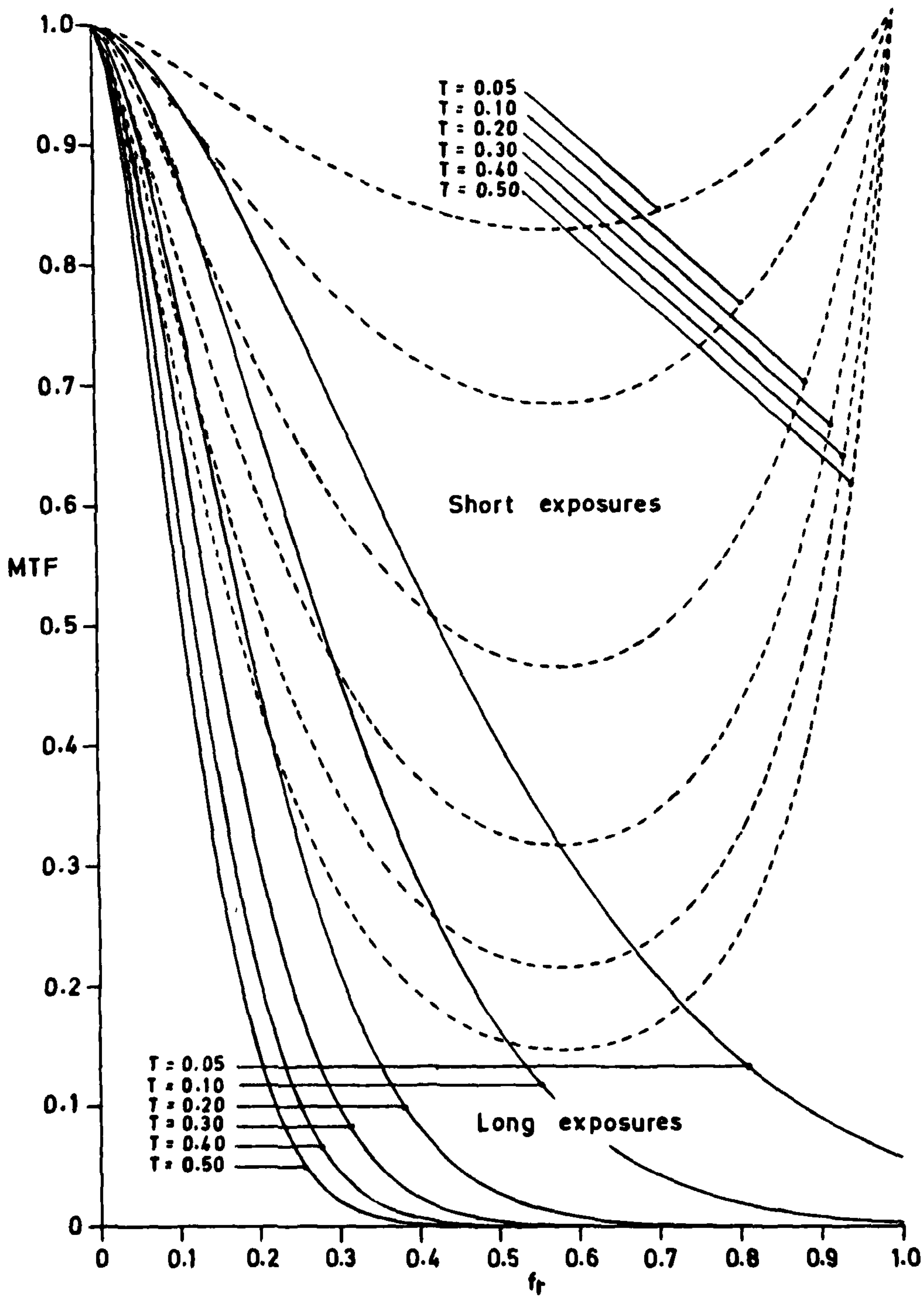

Fig 2 Turbulence 'MTF' factor

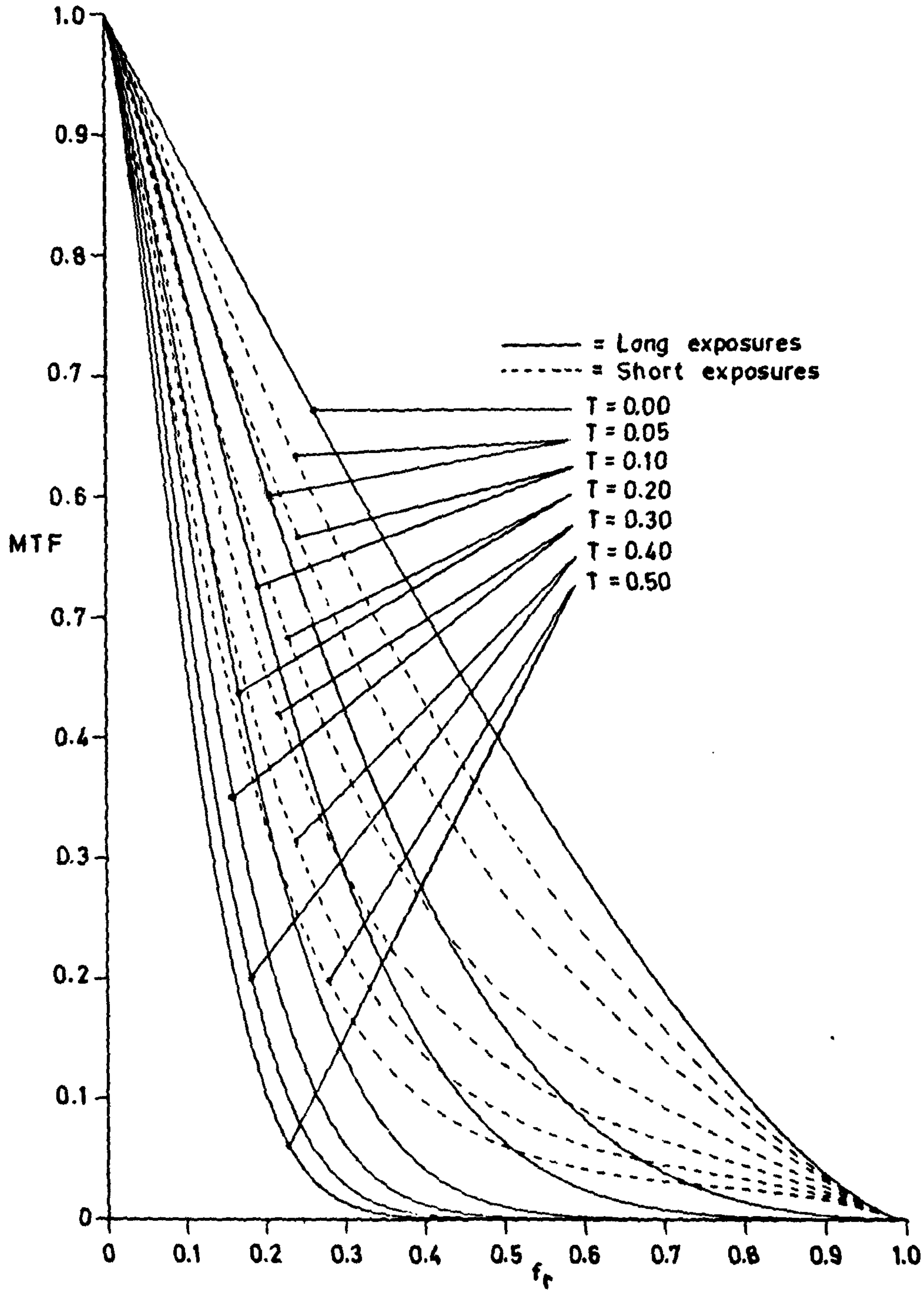

Fig 3 MTF of ideal lens and turbulence

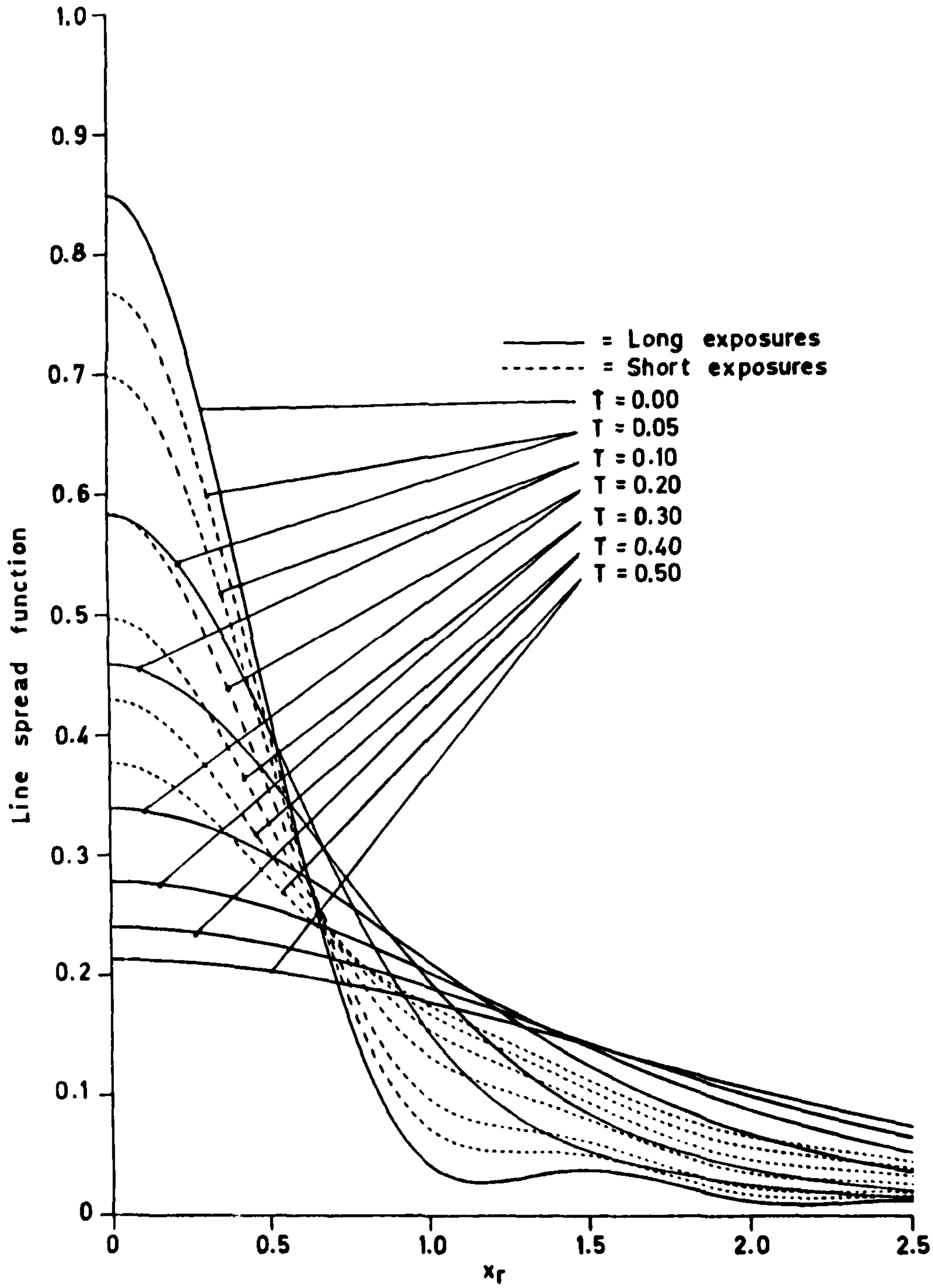

Fig 4 LSF of ideal lens and turbulence

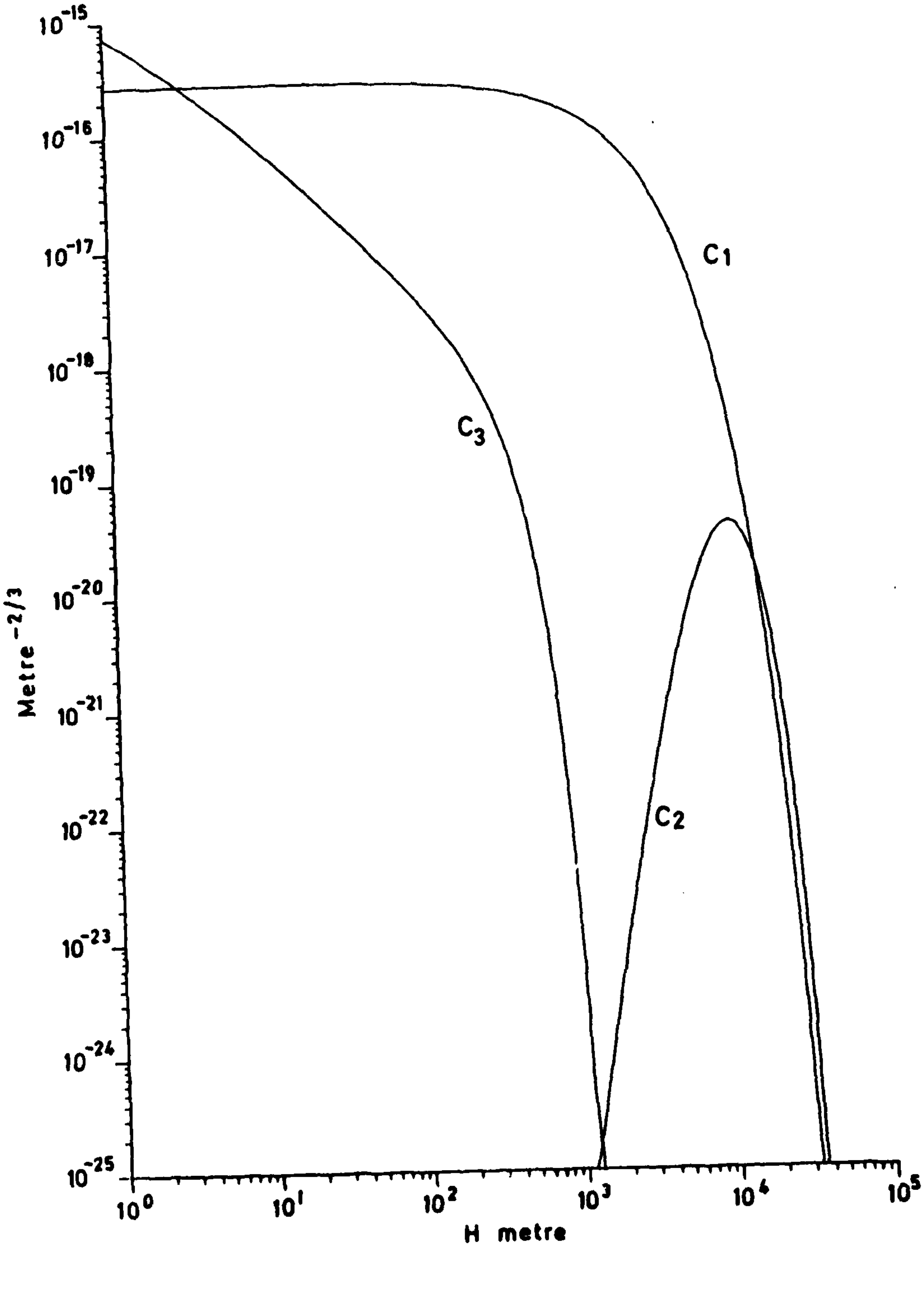

Fig 5 Coefficients of C_n^2

Fig 6

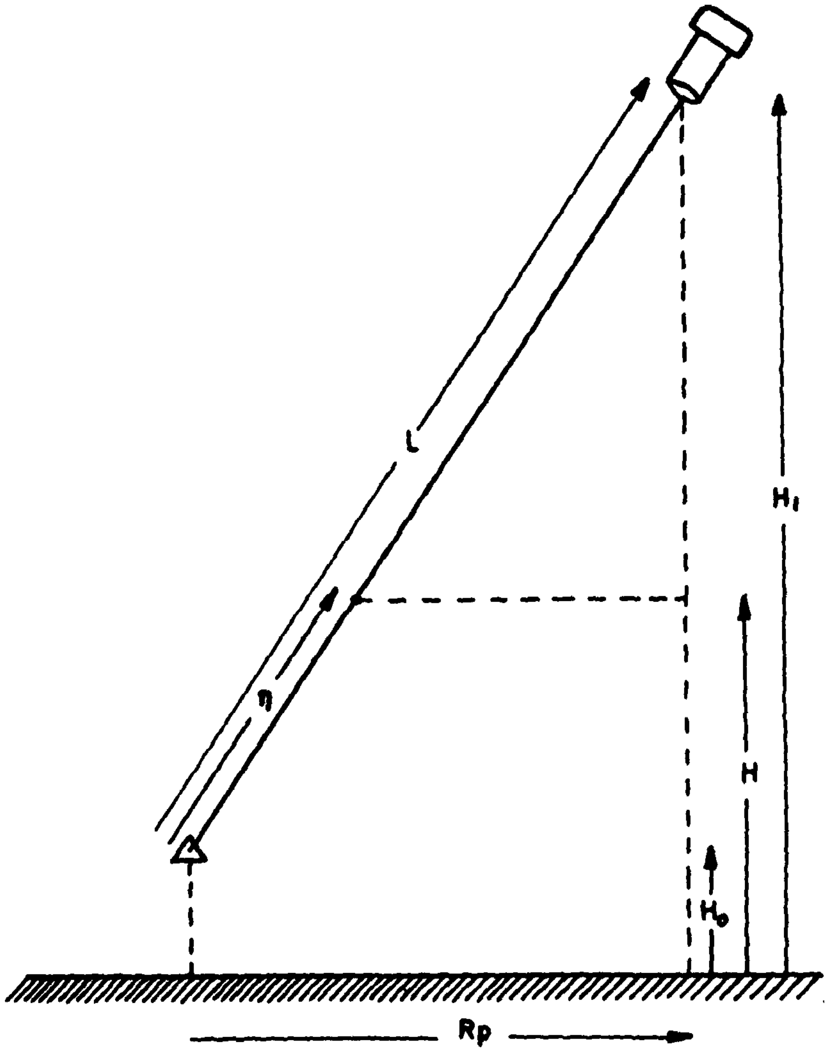

Fig 6 Geometry of optical path

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	RAE TR 81065	N/A	UNCLASSIFIED UNLIMITED

5. DRIC Code for Originator	6. Originator (Corporate Author) Name and Location
7673000W	Royal Aircraft Establishment, Farnborough, Hants, UK
5a. Sponsoring Agency's Code	**6a. Sponsoring Agency (Contract Authority) Name and Location**
N/A	N/A

7. Title The effect of atmospheric turbulence on air to ground photography

7a. (For Translations) Title in Foreign Language

7b. (For Conference Papers) Title, Place and Date of Conference

8. Author 1. Surname, Initials	9a. Author 2	9b. Authors 3, 4	10. Date	Pages	Refs.
Ballantyne, J.G.			May 1981	29	24
11. Contract Number	**12. Period**	**13. Project**	**14. Other Reference Nos.**		
N/A	N/A		IT 175		

15. Distribution statement

(a) Controlled by –

(b) Special limitations (if any) –

16. Descriptors (Keywords) (Descriptors marked * are selected from TEST)

Aerial photography*. Turbulence*.

17. Abstract

The current state of the theory of optical effects of atmospheric turbulence is summarized, and the derivation of an equation relating the modulation transfer function of an optical path through the atmosphere to a weighted integral of the structure constant of refractive index along that path is outlined.

Methods of estimating the structure constant from more readily available meteorological parameters are discussed, and FORTRAN subroutines for evaluating the integral and estimating the modulation transfer function of the path under any required conditions are given.

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